What DCC-GARCH Model Tell us About the Effect of the Gold Price’s Volatility on South African Exchange Rate?

By Leleng KEBALO †

Abstract. The aim of this paper is to study through a model rarely used and little known, the effect of the gold price’s volatility on the south african real exchange rate. More precisely, it is to show that, through the dynamic conditional correlation (DCC) GARCH model; we get results that are consistent with economic works (Frankel, 2007) on the relationship between gold price’s volatility and the real exchange rate. The period retained in this research paper going from May 1995 to April 2014 and the frequency of the data is monthly. After analysis, we find that in the short term, the real exchange rate is more sensitive to its own volatility, compared to the effect of the volatility of gold price. This last effect, although high, is less persistent on the real exchange rate.

Keywords. Volatility, Exchange rate, Gold price, DCC-GARCH.

JEL. C50, F30, F40.

1. Introduction

The relationship between economic fundamentals and the exchange rate regime is one of the most controversial issues of the international economy and a long-standing puzzle. The existence or not of a possible relationship is what on which economists focus theirs works. The exchange rate is a very important economic variable because it has some influence on the terms of trade, especially for major exporters of raw materials, energy or not. However, for countries whose economies are heavily dependent in exports of commodities, world prices seem to have a strong and systemic influence on their currencies (Chen & Rogoff, 2003). This is the case of some countries in America (Peru…), Africa (Democratic Republic of Congo, Angola…), eastern Europe and Asia (China Republic…).

One of the largest exporter of strategic raw materials in the world is the South Africa and the country has been the first african economic power before the Nigeriasince 2014. The country is favorable to the market economy and above all characterized by a good financial integration. The gross income of South Africa before 2013, represented the quarter (1/4) of the african grossdomestic product with an annual average rate of 5%. In addition to have the most competitive markets in Africa, the country has a strong endowment of raw materials such as gold, diamond, titanium, platinum… Among this resources basket, gold remains the first wealth of the country and a key impulse of the South African economy. South Africa has been also the world leader in the gold market until 2006; resource that the country has around 50% and 70% of planetary reserves (Source: World Gold

† University of Lome, Togo.

. kebalo.leleng@hotmail.fr
Journal of Economics Library

Since 2007, South Africa has lost his place of leader in term of production in favor of China at the time of the financial crisis.

In finance, gold has a particular quality that enhances risk management and capital preservation for institutional and private investors, more precisely during period of financial crisis. That is what the specialists on financial questions call the Safe haven or “Valeur refuge”. It is known that; a modest allocation of gold makes a valuable contribution to the performance of a portfolio by protecting it against downside risk without reducing the long-term returns. These qualities are especially considered during periods of financial instability and represents an opportunity for gold exporters and investors. An increasing in the price of gold represents a gain for the country, so a funding source of their economic activity. However, a declining, a shortfall for the country. Thus, all commodities prices fluctuations affect the real value of the currency. Recently, in South Africa, policymakers have found that the periods of high volatility of commodities prices have limited the investment and its effects were compounded in the periods when the South African Rand was overvalued with Dutch disease effects. Indeed, “given the high level of volatility in commodities prices, it is important for countries rich in natural resources in general, to understand well the relationship between the volatility of commodity prices and fluctuations in the rate of change” (Arezki et al., 2014).

In this paper, our main objective is to study through a model rarely used and little known, the effect of the volatility in gold price on the South African real exchange rate. More precisely, it is to show that, through the dynamic conditional correlation GARCH model - model used in our paper-, we get results that are consistent with economic works on the effects of gold price’s volatility shock on the real exchange rate. The period retained in this research paper going from May 1995 to April 2014, and the frequency of the data is monthly. After analysis, we find that in the short term, the real exchange rate is more sensitive to its own volatility, compared to the effect of volatility of gold price. This last effect, although high, is less persistent on the real exchange rate.

The rest of the paper is organized as follows: the section 2 defines the key concepts of our paper and perform a small literature review related to the research question. The section 3 presents the methodology; then the section 4 the data used, and after the section 5 presents the empirical results. Finally, the section 6 concludes our work.

2. Understand the volatility

2.1. Definition and some concepts

The volatility in general, measures the amplification of the variation of the price of an asset, a commodity or a financial variable. In others words, the volatility is the propensity of an asset to deviate from its average price over a given period. It is the intensity of the value of any asset around a trend. In terms of properties, the volatility varies over the time (heteroskedastic) and is self-correlated in majority (non-stationary). Most economic studies have shown that financial assets/variables are mostly heteroskedastic and past volatility influences the present one (Bollerslev, 1990; Engle, 2002; Engle & Kroner, 1995). In addition, the volatility of an asset is grouped in packets, i.e. there is a succession of phases of low volatility followed by high volatility ones and vice versa (Clusters Volatility). Besides this, it is important to note that in the analysis of the volatility, downturns assets tend to generate heavy volatility than those induced by the same magnitude of the increasing of price: that is called the Leverage effect.
The volatility analysis plays an important role because it helps to take economic and financial decisions. The volatility analysis arouses the interest of investors, policy makers, because it is both a source of risk and opportunity. In short term, high volatility equals to a higher likelihood of losses, so a possibility that the asset price collapses strongly and quickly. For an investor of long-term, volatility is the opportunity to buy when prices are low and sell when prices are too high, so an opportunity to gain more. This means that, the expected return is greater when the asset sees his price climb over quickly and strongly ("The risk is high; the expected gain will be ").

2.2. Volatility transmission models

In our study, we are interested in a specific model to analyze the transmission of volatility. Most of the time, the models used to analyze the transmission of volatility are the threshold and GARCH\(^1\) models. These last one is the king of models that we will use in our work. But we have two type of GARCH models. The univariate and multivariate models. What is the difference between the two models? The univariate GARCH models are more interested to analyze the sensitivity and the persistence of the volatility of a variable on itself. Conversely, multivariate GARCH models (MGARCH) analyze the impact of the volatility of a variable on another variable. The most obvious application of MGARCH models is the study of the relationships between co-volatilities and volatilities in a several market. In our study we analyze the effect of the volatility of gold’s price on the South African real exchange rate.

However, it is important to say why we use in our study the MGARCH model. The MGARCH model is rarely used to estimate this kind of relationship in economy than in finance; but very effective because it takes fully account of the price’s volatility, the non-linearity in variance, especially heteroskedasticity. In addition, these models can firstly help to perform excellent macroeconomic forecasting and secondly to take good economic decisions that are consistent with the literature. Apergis & Papoulakos (2013) through their study of the Australian dollar and the gold price showed the importance of using the multivariate GARCH models. By using variables such as terms of trade, the differential in interest rates, gold prices and the real exchange rate, they find that the conditional volatility of the exchange rate incorporates some information on the price of gold, i.e., the conditional volatility of the exchange rate may help to predict the future price of gold. These empirical results provide a support to the literature of Meese & Rogoff (1983), on the ability of the volatility of exchange rate to predict the future changes in commodity prices.

3. Methodology

If the univariate case is the subject of many studies, the multivariate case is still little studied. In this section we define the Dynamic Conditional Correlation GARCH model introduced by Engle (2002). Note that in practice, the fact to analyze only one variable is not very useful. The interest of a study based on the transmission of volatility is to review and analyze the various relationships that have different series together. Thus, understanding and predicting the time dependence in moments of second order of an asset is important in financial econometrics. It is now widely accepted that financial volatility moves in time through the assets and markets. Recognizing this function through a multivariate modeling framework led to more relevant empirical models than working with

\(^1\) Generalized autoregressive conditional heteroscedasticity
separate univariate models. Therefore, analyze the transmission of volatility through financial market variables requires the use of multivariate GARCH models.

Before performed any analysis by multivariate GARCH approach, it is important to verify the presence of GARCH effect at least in the explanatory variable. The test performed is the Ljung-Box test based on the squares of the returns of each series. This is a matching test; a non-correlation test on the square of returns. The null hypothesis of no autocorrelation in the square of yields corresponds to the existence of ARCH effects in the variable. However, the alternative hypothesis of the presence of autocorrelation corresponds to the existence of GARCH effect in the variable. For the orders \( p \) and \( q \), a Box-Jenkins selection procedure is used. The maximum likelihood method is used to estimate the GARCH model.

Let define a DCC-GARCH model. Let \( X_t \) a vector \((n \times 1)\) of stationary process, \( X_t \sim DCC - GARCH \) if:

\[
X_t = \mu_t + \varepsilon_t \\
\varepsilon_t = H_t^{1/2} \theta_t \\
H_t = D_t R_t D_t
\]

with

\[
D_t = \begin{pmatrix}
\sqrt{H_{1,t}} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \sqrt{H_{n,t}}
\end{pmatrix}
\]

and

\[
R_t = \begin{pmatrix}
1 & \rho_{12,t} & \rho_{13,t} & \cdots & \rho_{1n,t} \\
\rho_{21,t} & 1 & \rho_{23,t} & \cdots & \rho_{2n,t} \\
\rho_{31} & \rho_{32,t} & 1 & \cdots & \vdots \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\rho_{n1,t} & \rho_{n2,t} & \cdots & \rho_{n,n-1,t} & 1
\end{pmatrix}
\]

where

\[
H_t = \alpha_0 + \sum_{q=1}^{Q_1} \alpha_q \theta_{t-q}^2 + \sum_{p=1}^{P} \beta_{ip} H_{t-p}
\]

\( \mu_t \): a vector \((n \times 1)\) of conditional expectation of \( X_t \) at \( t \),

\( \varepsilon_t \): a vector \((n \times 1)\) conditionals errors of \( n \) series at \( t \), with \( E(\varepsilon_t) = 0 \) and \( \text{Cov}(\varepsilon_t) = H_t \).

\( H_t \): is the matrix \((n \times n)\) of conditional variance and covariance of \( \varepsilon_t \) at \( t \);

\( D_t \): is the diagonal matrix \((n \times n)\) of conditional standard errors of \( \varepsilon_t \) at \( t \), which is always positive;

\( R_t \): is the matrix \((n \times n)\) of conditional correlations of \( \varepsilon_t \) at \( t \).

\( \theta_t \): a vector \((n \times 1)\) of errors i.i.d. with \( E(\varepsilon_t) = 0 \) and \( E(\varepsilon_t, \varepsilon_t') = I_n \)

We note that \( R_t \) is the dynamic matrix. \( H_t \) must be always positive. \( R_t \) should be positive and also that its elements are less than or equal to one \((\rho_i \leq 1 \ \forall \ i)\). For this we break \( R_t \) in two (02) matrix:

\[
R_t = Q_t^{t-1} Q_t Q_t^{t-1}
\]
with
\[ Q_t^* = \begin{pmatrix} \sqrt{q_{11,t}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sqrt{q_{nn,t}} \end{pmatrix} \]

and
\[ Q_t = \begin{pmatrix} q_{11,t} & \sqrt{q_{11,t}q_{22,t}} & \cdots & \sqrt{q_{11,t}q_{nn,t}} \\ \sqrt{q_{11,t}q_{nn,t}} & q_{22,t} & \cdots & \sqrt{q_{22,t}q_{nn,t}} \\ \vdots & \vdots & \ddots & \vdots \\ \sqrt{q_{11,t}q_{nn,t}} & \sqrt{q_{22,t}q_{nn,t}} & \cdots & q_{nn,t} \end{pmatrix} \]

For a DCC(p,q)
\[ Q_t = \left( 1 - \sum_{i=1}^{p} \sum_{j=1}^{q} \alpha_{DCC,i,j} - \sum_{j=1}^{q} \beta_{DCC,j} \right) \bar{Q} + \sum_{i=1}^{p} \sum_{j=1}^{q} \alpha_{DCC,i,j} (\varepsilon_{t-i} \varepsilon_{t-i}') + \sum_{j=1}^{q} \beta_{DCC,j} Q_{t-j} \]

with
\[ \bar{Q} = E(\varepsilon_t \varepsilon_t') \]

For a DCC(1,1)
\[ Q_t = \left( 1 - \alpha_{DCC} - \beta_{DCC} \right) \bar{Q} + \alpha_{DCC} \varepsilon_{t-1} \varepsilon_{t-1}' + \beta_{DCC} Q_{t-1} \]

with \( \alpha_{DCC} \) et \( \beta_{DCC} \) are scalars. \( R_t > 0 \) if and only if \( Q_t > 0 \). So that \( H_t \) is positive, it is necessary that the following conditions are satisfied:
\[ \alpha_{DCC} \geq 0; \beta_{DCC} \geq 0; (\alpha_{DCC} + \beta_{DCC}) < 1 \]

Consider a DCC-GARCH (1,1) bivariate model used in this study:
\[ H_{11,t} = \alpha_{0,1} + \alpha_{11} \varepsilon_{t-1}^2 + \beta_{11} H_{11,t-1} \]
\[ H_{22,t} = \alpha_{0,2} + \alpha_{21} \varepsilon_{t-1}^2 + \beta_{21} H_{22,t-1} \]
\[ Q_t = \left( 1 - \alpha_{DCC} - \beta_{DCC} \right) \bar{Q} + \alpha_{DCC} \varepsilon_{t-1} \varepsilon_{t-1}' + \beta_{DCC} Q_{t-1} \]

The parameters to be estimated are \( \alpha_{0,1,2} \), representing respectively the average conditional volatility of series 1 et 2; \( \alpha_{11,21} \) called ARCH parameters measure the sensitivity and \( \beta_{11,21} \) GARCH parameters, measure the persistence. Thus \( \alpha_{i,j} \forall i,j = 1,2 \) measure the sensitivity of the volatility of the asset i on the asset j and \( \beta_{i,j} \forall i,j = 1,2 \) measure the persistence of the shock of asset i on the asset j.

The estimate of the DCC-GARCH model is performed by the maximum likelihood method and the likelihood function for \( X_t = \sqrt{H_t} \varepsilon_t \) is written:
\[ L(\theta) = \prod_{t=1}^{T} \frac{1}{(\sqrt{2\pi})^n/H_t} \exp \left( -\frac{1}{2} X_t^T H_t X_t \right) \]

JEL, 3(4), L. Kebalo, p.570-582.
where \( \theta = (\phi, \omega) \) are the parameters to be estimated;

with

\[
\phi = (\alpha_0, \alpha_1, \ldots, \alpha_p, \beta_1, \ldots, \beta_Q)
\]

and

\[
\omega = (\alpha_{DCC}, \beta_{DCC})
\]

The log-likelihood is written as follows:

\[
L = -\frac{1}{2} \sum_{t=1}^{T} \left( n\log(2\pi) + \log|D_t| + X_t^2 X_t^{-1} X_t \right),
\]

\[
L = -\frac{1}{2} \sum_{t=1}^{T} \left( n\log(2\pi) + 2\log|D_t| + \log|R_t| + X_t^2 R_t^{-1} R_t^{-1} D_t^{-1} X_t \right)
\]

The log-likelihood is the sum of a term of volatility \( L_v(\theta) \) and another of correlation \( L_c(\theta, \varnothing) \)

\[
L(\theta, \varnothing) = L_v(\theta) + L_c(\theta, \varnothing)
\]

with

\[
L_v(\theta) = -\frac{1}{2} \sum_{t=1}^{T} \left( n\log(2\pi) + 2\log|D_t| + X_t^2 R_t^{-1} R_t^{-1} D_t^{-1} X_t \right)
\]

and

\[
L_c(\theta, \varnothing) = -\frac{1}{2} \sum_{t=1}^{T} \left( \log|R_t| + \epsilon_t^2 R_t^{-1} \epsilon_t^2 - \epsilon_t^2 \right)
\]

The model estimation is done in two steps. The first is to estimate the conditional variance of series with a univariate GARCH \( L_v(\theta) \), and the second is to use the standardized residuals obtained to estimate the parameters of the dynamic correlation matrix \( L_c(\theta, \varnothing) \).

First step: In this step we maximize the term of volatility \( L_v(\theta) \). We replace \( R_t \) by an identity matrix \( I_n \). So we have:

\[
L_v(\theta) = -\frac{1}{2} \sum_{t=1}^{T} \left( n\log(2\pi) + 2\log|D_t| + \log|I_n| + X_t^2 D_t^{-1} R_t^{-1} D_t^{-1} X_t \right)
\]

\[
= -\frac{1}{2} \sum_{t=1}^{T} \left( n\log(2\pi) + 2\log|D_t| + X_t^2 D_t^{-1} R_t^{-1} D_t^{-1} X_t \right)
\]

\[
= -\frac{1}{2} \sum_{t=1}^{T} \left( n\log(2\pi) + \sum_{i=1}^{n} \left( \log(H_{it}) + \frac{X_{it}^2}{H_{it}} \right) \right)
\]

\[
= -\frac{1}{2} \sum_{t=1}^{T} \sum_{i=1}^{n} \left( \log(2\pi) + \log(H_{it}) + \frac{X_{it}^2}{H_{it}} \right)
\]

\[
\log(2\pi) \text{ is constant, so we maximize:}
\]
The estimator \( \theta^* \) of \( \theta \) is obtained by:

\[
\theta^* = \arg_{\theta} \max_{\theta} L_{v}^{\theta} (\theta)
\]

**Second step:** This step consists to estimate the correlation term \( L_c(\theta, \varnothing) \). Once the first step is completed, this is performed using the likelihood function now well specified:

\[
L_c(\theta, \varnothing) = -\frac{1}{2} \sum_{t=1}^{T} \left( n \log(2\pi) + 2 \log |D_t| + \log |R_t| + X_t D_t^{-1} R_t^{-1} D_t^{-1} X_t \right).
\]

and we obtain the estimator:

\[
\varnothing^* = \arg_{\varnothing} \max_{\varnothing} L_c(\theta, \varnothing)
\]

Under general conditions, the likelihood estimator will converge and will be asymptotically normal:

\[
\sqrt{T}(\hat{\theta} - \theta_0) \overset{d}{\rightarrow} \mathcal{N}(0, V(\theta_0))
\]

In terms of benefits, the DCC-GARCH models are modeling directly the variance and the covariance but also its flexibility. They allow to take into account the change in the relationship between volatility of variables over the time, so to measure the real impact of the volatility of a variable on another.

### 4. Data

After defining the model, we present the variables used in our study (see Table 1). The data considered in our study are monthly and the period goes from May 1995 to March 2014. Before any statistical operation, we remove the seasonal effect in the series of gold prices “GP” with the x12-arima process. Once done, we define two variables - the real exchange rate and the real gold price - that we need for our analysis.

**Table 1. Series**

<table>
<thead>
<tr>
<th>Name</th>
<th>Label</th>
<th>Source</th>
<th>Seasonality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold price</td>
<td>GP</td>
<td>Federal reserve of St Louis</td>
<td>Yes</td>
</tr>
<tr>
<td>Nominal exchange rate</td>
<td>S</td>
<td>OECD stats</td>
<td>No</td>
</tr>
<tr>
<td>South Africa CPI</td>
<td>( \bar{p}_{sa} )</td>
<td>OECD stats</td>
<td>No</td>
</tr>
<tr>
<td>United states CPI</td>
<td>( \bar{p}_{us} )</td>
<td>OECD stats</td>
<td>No</td>
</tr>
</tbody>
</table>

*Note: CPI for consumer price index*

#### 4.1. Real exchange rate

Figure 1 shows the evolution of the real exchange rate calculated as follows:
with the nominal exchange rate (price quotation system), $p_{t}^{US}$ the consumer price index of United States, and $p_{t}^{SA}$ the consumer price index of South Africa. All variables are expressed in logarithms. The analysis of the graph presents much transitory shocks to the real value of the Rand. The most salient points are the gradual depreciation of the Rand which lasted from 1999 to 2002 (the sharp depreciation of the Rand), and also from 2007 to 2008 during financial crisis. Careful analysis of the graph shows a cyclical evolution of the exchange rate, cycles which tend to decline over the years.

\[ r_{ert} = s_{t} + p_{t}^{US} - p_{t}^{SA} \]

\[ \text{Figure 1. South african real exchange rate (in logs)} \]

4.2. Price of gold
For the evolution of world gold price shown in figure 2, apart the slight decrease in the price of gold between 1995 and 2002, the value of the gold has increased steadily by attending an historic price of 1,896.5 dollars per troy ounce on September 2011. Since the value of the gold has steadily declined. But it is important to note the slight fall of the value of gold (692.50 USD) during the 2008 recession before increases thereafter.

\[ \text{Figure 2. Evolution of price of gold} \]

4.3. Joint analysis
When we jointly analyze the evolution of the two series, we note that the decline of gold price is followed by a depreciation of the exchange rate. Thus, after the recession of 2001, we note that the rise in world gold price until 2008 is followed by a phase of appreciation of the South African currency. The slight drop in the price of gold in November 2008 corresponds to a real depreciation of the Rand. Thus, there is therefore a correspondence lower world price of gold -real depreciation of the exchange rate on one side, and rising global gold prices- real

JEL, 3(4), L. Kebalo, p.570-582.
appreciation of the exchange rate” on the other hand. What seems normal for a coherent and exporter of raw materials.

4.4. Unit root test

We perform only one unit root test to verify the order of integration of the two series. The Augmented Dickey-Fuller test tells us that the two variables are integrated of order 1 because our two variables vary over time and do not seem stationary. To make them stationary and usable for our study, we differentiate the two series.

Table 2. Unit root test

<table>
<thead>
<tr>
<th></th>
<th>ADF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level</td>
</tr>
<tr>
<td>Rer</td>
<td>I(1)</td>
</tr>
<tr>
<td>Gp</td>
<td>I(1)</td>
</tr>
</tbody>
</table>

Once the unit root test performed (see Appendix), we can now focus our empirical results.

5. Empirical Results

5.1. Stability test

Before analyzing the volatility, it is necessary to verify the stability of the relationship between the real exchange rate and the price of gold being over the period. July 2007, date of the beginning of the global financial crisis -marked by a liquidity crisis- represents the breaking point of our analysis. The stability test performed indicates that the link between the real exchange rate and the price of gold is stable around the period. The probability associated with the test(Fisher) 0.9172 is above the threshold of $\alpha= 5\%$, which does not allow us to reject the hypothesis of relationship stability.

Table 3. Stability test of Chow

<table>
<thead>
<tr>
<th>Stability test</th>
<th>F-Statistic</th>
<th>Prob. F(2, 223)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-Statistic</td>
<td>0.086420</td>
<td>0.9172</td>
</tr>
</tbody>
</table>

Note: Chow breakpoint test: July 2007
Null hypothesis: No breaks at specified breakpoints

We cannot perform our analysis with MGARCH whether residues between the real exchange rate and the gold price are not heteroskedastic. The White test performed and presented in table 4 shows that the variance of the residuals between the two variables varies over time. The volatility varies over time.

Table 4. VAR Residual Heteroskedasticity Tests: No Cross Terms

<table>
<thead>
<tr>
<th>Chi-sq</th>
<th>df</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>31.71035</td>
<td>10</td>
<td>0.0015</td>
</tr>
</tbody>
</table>

5.2. DCC-GARCH(1,1) model

We study the sensitivity and the persistence through a DCC-GARCH model. This, allows us to measure the real impact of gold price’s volatility on the south african real exchange rate. This model allows to take into account the dynamics of the volatility and the correlation between the two variables. We start from a VAR (1) model\(^2\) to define our GARCH model:

\[^2p = 1\] is the number of Lag allowing to have non-autocorrelated residuals
Journal of Economics Library

\[
\begin{aligned}
DLRER_t &= \alpha_1 + \sum_{i=1}^{5} \beta_{11} DLRER_{t-i} + \sum_{j=1}^{5} \delta_{1j} DLGP_{t-j} + \varepsilon_{1t} \\
DLGP_t &= \alpha_2 + \sum_{i=1}^{5} \beta_{21} DLRER_{t-i} + \sum_{j=1}^{5} \delta_{2j} DLGP_{t-j} + \varepsilon_{2t}
\end{aligned}
\]

with

\[
\begin{aligned}
\varepsilon_t &= H_t^{\frac{1}{2}} \varepsilon_t \\
H_t &= D_t R_t D_t \\
H_{11,t} &= c_1 + a_{11} \varepsilon_{1,t-1}^2 + b_{11} H_{11,t-1} \\
H_{22,t} &= c_2 + a_{21} \varepsilon_{2,t-1}^2 + b_{21} H_{22,t-1} \\
Q_t &= (1 - \alpha_{DCC} - \beta_{DCC}) \bar{Q} + \alpha_{DCC} \varepsilon_{1,t-1} + \beta_{DCC} Q_{t-1} \\
\alpha_{DCC} &\geq 0; \quad \beta_{DCC} \geq 0; \quad (\alpha_{DCC} + \beta_{DCC}) < 1.
\end{aligned}
\]

with

\[
D_t = \begin{pmatrix}
\sqrt{H_{11,t}} & 0 \\
0 & \sqrt{H_{22,t}}
\end{pmatrix},
\]

and

\[
R_t = \begin{pmatrix}
1 & \rho_{12,t} \\
\rho_{21,t} & 1
\end{pmatrix},
\]

\[
\varepsilon_t = (\varepsilon_{1,t}, \varepsilon_{2,t}), \text{ with } E(\varepsilon_t) = 0 \text{ et } Cov(\varepsilon_t) = H_t. \varepsilon_t \text{ is the error vector i.i.d. H}_t \text{ is the conditional variance-covariance matrix. The parameters of interest are:} \theta = (\alpha_{0,1}, \alpha_{1,1}, \beta_{11}, \alpha_{0,2}, \alpha_{21}, \beta_{21}, \alpha_{DCC}, \beta_{DCC}). \text{ The assets 1 and 2 represent the real exchange rate and the gold price. The estimation of the model by the maximum likelihood method gives the results shown in the table 5.}
\]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std Error</th>
<th>T-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{0,1}$</td>
<td>0.0000126</td>
<td>0.000023</td>
<td>5.58100</td>
</tr>
<tr>
<td>$\alpha_{0,2}$</td>
<td>0.000296</td>
<td>0.000023</td>
<td>12.64768</td>
</tr>
<tr>
<td>$\alpha_{11}$</td>
<td>0.268775</td>
<td>0.023349</td>
<td>11.51121</td>
</tr>
<tr>
<td>$\beta_{11}$</td>
<td>0.688840</td>
<td>0.016523</td>
<td>41.69050</td>
</tr>
<tr>
<td>$\alpha_{21}$</td>
<td>0.174110</td>
<td>0.037532</td>
<td>4.63893</td>
</tr>
<tr>
<td>$\beta_{21}$</td>
<td>0.585653</td>
<td>0.022692</td>
<td>25.80881</td>
</tr>
<tr>
<td>$\alpha_{DCC}$</td>
<td>0.000000</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\beta_{DCC}$</td>
<td>0.543939</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Our results are consistent with those of the theory. The coefficient $\alpha_{DCC}$ is approximatively equal to zero, the coefficient $\beta_{DCC}$ greater than zero, and the sum of two which is less than 1 ($\alpha_{DCC} + \beta_{DCC} < 1$). However, before the interpretation of our results, it is necessary to validate the results of our model. This need the stationarity of our residuals.
The figure 3 shows us that the mean of residual series seems constant along the time and seem stationary. Results which allows us to validate our model.

5.3. Interpretation of results

Remember that $\alpha_{ij}$ indicates the sensitivity of the asset $j$ following a volatility shock of the asset $i$. $\beta_{ij}$ indicates the persistence of the Asset $j$ following a volatility shock of the asset $i$. We interpret the results presented in Table 5.

The volatility of the real exchange rate and the price of gold are on average close to zero: 0.000126 for the real exchange rate and 0.000296 for gold price. The phases of real appreciation of the real exchange rate and rising in gold price are more recurrent in the evolution of both series. In addition, the South African Rand is more sensitive to its own volatility shock (0.268775) compared to the volatility shock of the price of gold (0.174110). As in Frankel (2007), the 2002 depreciations phases are not explained by decreases in the price of gold. Regarding the persistence of shock, we find that the impact of the volatility in the real exchange rate on itself are more persistent and amounts to 68.88%, compared to the persistence of volatility shocks of the price of gold on the actual value of the Rand which amounts to 58.565%. More precisely, our results show that the real exchange rate is influenced by its own volatility. Gold price volatility’s shocks have a persistent effect on the real value of the rand, however this effect is less important than the effect thereal exchange rate. It is important to clarify that the persistence of the gold price volatility shocks on the exchange rate is high also.

6. Conclusion

We set ourselves the aim of studying through a model rarely used and little known, the effect of the volatility of the price of gold on the South African real exchange rate over the period from May 1995 to April 2014. More precisely, it is to show on one hand that, the volatility of the price of gold influencesthe south african real exchange rate, but on the other hand to show that through the dynamic conditional correlations GARCH model, results are consistent with those of economic work (Frankel, 2007). After analysis, we find that, the real exchange rate of the South African Rand is more sensitive to its own volatility relatively to the impact of the volatility of the price of gold. The effect of volatility of the price of gold, even high, are less persistent on the real exchange rate, than the effect of the volatility of exchange rate on itself.
### Appendix

**Table 6. Unit Root test**

<table>
<thead>
<tr>
<th></th>
<th>ADF unit root test</th>
<th>Phillips-Perron unit root test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level</td>
<td>First difference</td>
</tr>
<tr>
<td>Rer</td>
<td>1.342277</td>
<td>-43.40724</td>
</tr>
<tr>
<td>Gp</td>
<td>1.720518</td>
<td>22.30633</td>
</tr>
</tbody>
</table>

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Rer</td>
<td>1.298114</td>
<td>-71.19146</td>
</tr>
<tr>
<td>Gp</td>
<td>1.534676</td>
<td>-73.65879</td>
</tr>
</tbody>
</table>
References

Copyrights
Copyright for this article is retained by the author(s), with first publication rights granted to the journal. This is an open-access article distributed under the terms and conditions of the Creative Commons Attribution license (http://creativecommons.org/licenses/by-nc/4.0).